

MASS FORMULAS FOR SINGLE-CHARM TETRAQUARKS WITH FERMI-BREIT HYPERFINE INTERACTION

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In this paper we present the main results of our investigation of the $cq\bar{q}\bar{q}$ single-charm scalar tetraquarks and their $SU(3)_F$ representations: $\bar{15}_S$, $\bar{3}_S$, 6_A and $\bar{3}_A$. We use the Fermi-Breit interaction Hamiltonian with $SU(3)$ flavor symmetry breaking to determine the masses of the single-charm tetraquarks. We also discuss mass spectra obtained from meson and baryon mass fits. The mass spectra are very similar to those obtained with Glozman-Riska hyperfine interaction, and they indicate that some of the experimentally detected states may have tetraquark nature.

Key words: nonrelativistic quark model, hadron mass models and calculations, light quarks, charmed quarks.

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1. INTRODUCTION

The possible existence of four-quark states for light flavor dimensions, as well as some predictions for tetraquark spectroscopy, was first suggested by Jaffe [1]. In Ref. [2] it is also provided a framework for a quark-model classification of the many two-quark-two-antiquark states. In Ref. [3] the energies of diquonia $q^2(\bar{q}^2)$ with orbital angular momentum $L = 0$ are calculated and compared to the threshold energies. The first observations of the scalar charmed mesons have been reported [4–8]. In Refs. [9–11] some observed mesons (as for instance $D_s^+(2317)$) are explained as a scalar $c\bar{s}$ systems. Many studies appeared in the past on $D_s^+(2317)$ ([12–14] and references therein). Possible interpretations include tetraquark states or molecular DK or DK^* states [15]. The origin of the lightest scalar mesons, in particular, the light σ -meson is given in [16] in the framework of the instanton liquid model of the QCD vacuum. Terasaki and Hayashigaki [17, 18] have investigated the decay rates of the members of the same multiplet in class of four-quark mesons. Also, in Refs. [19–21] it is shown that the existence of some exotic states with the four-quark structures might be expected. There are more results indicating that a diquark-antidiquark structure is acceptable for some observed states [22–26]. In Refs. [27, 28] a mixture of conventional quark-antiquark states and four-quark components is considered. Also,

there are predictions for some hidden charm states to be tetraquarks. For example, in [29] the masses of the ground state heavy tetraquarks are calculated in the framework of the relativistic quark model, based on the quasipotential approach in quantum chromodynamics. These authors found that some exotic meson candidates can be tetraquark states with hidden charm. A concise overview of mesons with heavy quarks including charmed mesons and charmonium (or charmonium-like) states is given in [30]. The mass spectrum of the scalar hidden charm and bottom tetraquark states is studied in [31].

In this paper we perform a schematic study (two quark interaction) of the masses of the single-charm $cq\bar{q}\bar{q}$ tetraquarks in the SU(3) flavor representations. We consider states with the spin-parity quantum numbers $J^P = 0^+$ (J = total angular momentum, P = parity). Using the colored version of the Fermi-Breit (FB) hyperfine interaction (HFI) [3, 32–35] we investigate the possible four-quark structure of these mesons. According to the mass constraint, experimentally indicated states can contain only one c -quark. If each of them contains no heavy quarks, mass is too low, and if it contains one heavy quark (b or t) or two c -quarks mass will be too high. Only two possible solutions are: $cq\bar{q}\bar{q}$ that we investigate and $\bar{c}\bar{q}qq$ that is analyzed by Liu et al. [23].

We showed [36] that the constituent quark masses are very sensitive to the system in which they are contained, and their values differ less or more in different systems. That is why we analyzed tetraquark masses using independent meson and baryon fits for constituent quark masses. These fits satisfy Feynman-Hellmann theorem for FB interaction [37, 38]. Constituent quark masses in tetraquarks are somewhat different for both cases: quark masses from meson fit and from baryon fit [36, 39].

We choose relatively simple FB HFI because it nicely satisfies Roncaglia inequalities for mass differences [37, 38]. We deal with the open charm states. Since our intention is to investigate the possible tetraquark nature of 27 $cq\bar{q}\bar{q}$ states, it is not necessary to develop an advanced relativistic quark model. That is why masses of the scalar charmed tetraquarks are discussed in the framework of the nonrelativistic quark model, in which the mass of a hadron is considered to be the sum of the constituent quark masses and contributions of the FB HFI. As it will be shown later, our simple model is quite sufficient for investigating wave functions and masses of these states. Also, this model is used to get estimates of the theoretical meson and baryon masses. We prefer to deal with four quarks instead of diquarks because our system under consideration consists of one heavy and three light quarks.

In this paper we present detailed calculation of masses using FB interaction, and it is the first time this interaction is applied to 27 states of $cq\bar{q}\bar{q}$ tetraquarks in order to derive formulas for masses. We also compare these results with another phenomenological interaction: Glozman-Riska hyperfine interaction (GR HFI) [40].

2. ANALYSIS AND METHOD

We discussed single-charm tetraquarks in Ref. [41], where the flavor wave functions and masses of $cq\bar{q}\bar{q}$ tetraquarks are calculated using GR HFI. Now we present detailed calculation of masses of $cq\bar{q}\bar{q}$ tetraquarks using FB HFI.

We analyze the tetraquark states with one charm quark which is singlet under the transformation of $SU(3)_F$. There are four multiplets according to product: $3 \otimes \bar{3} \otimes \bar{3} \otimes 1 = \bar{15}_S + \bar{3}_S + 6_A + \bar{3}_A$, i.e. two anti-triplets, one anti-15-plet and one sextet. Young diagrams for these $SU(3)_F$ multiplets, as well as the weight diagrams, can be found in Figs. 1 - 5 of [41]. In these tetraquark multiplets, all states have a charm number equal to 1. Labels for all 27 states are the same as in Refs. [41, 42]. These labels are taken only by analogy with baryons, but of course they are not baryons. Many authors use the same labeling [19, 20]. The strong FB HFI Hamiltonian [3] may be written in the following form:

$$H_{FB} = C \sum_{i>j=1}^4 \frac{\vec{\sigma}_i \vec{\sigma}_j}{m_i m_j} (\lambda_i^C \lambda_j^C), \quad (1)$$

where σ_i are the Pauli spin matrices, λ_i^C are the color Gell-Mann matrices and C is a constant proportional to strong hyperfine structure constant α_c . Hamiltonian (1) has explicit color and spin exchange dependence and implicit (by way of quark masses) flavor dependence. Its contribution to tetraquark masses is:

$$m_{\nu, FB} = \langle \nu | \langle \chi | H_{FB} | \chi \rangle | \nu \rangle, \quad (2)$$

where χ denotes the spin wave function and ν - flavor wave function. For total masses m_ν we have:

$$m_\nu = m_{\nu, 0} + m_{\nu, FB}, \quad (3)$$

where $m_{\nu, 0}$ are masses without influence of FB HFI.

Here we have to mention that the mixing of states is taken into account. In Table I of the paper [41] the four-quark content and quantum numbers of scalar $cq\bar{q}\bar{q}$ tetraquarks are given. One can see from that table which states mix due to the same quantum numbers. So we took into account the mixing when calculating masses of these tetraquarks.

3. RESULTS AND DISCUSSION

In our model (total spin = 0), the corresponding symmetric χ_S and antisymmetric χ_A spin functions have the following forms:

$$|\chi_S\rangle = \frac{-1}{2\sqrt{3}} |\uparrow\downarrow\uparrow\downarrow + \downarrow\uparrow\uparrow\downarrow + \uparrow\downarrow\downarrow\uparrow + \downarrow\uparrow\downarrow\uparrow - 2\uparrow\uparrow\downarrow\downarrow - 2\downarrow\downarrow\uparrow\uparrow\rangle, \quad (4)$$

$$|\chi_A\rangle = \frac{1}{2} |\uparrow\downarrow\uparrow\downarrow - \downarrow\uparrow\uparrow\downarrow - \uparrow\downarrow\downarrow\uparrow + \downarrow\uparrow\downarrow\uparrow\rangle. \quad (5)$$

The spin wave functions are symmetric (χ_S) or antisymmetric (χ_A) under interchange between the pair of quarks and between the pair of antiquarks (not for quark-antiquark interchange). Tetraquarks are bosons, i.e. they have integer spin (for scalar tetraquarks it equals 0). For the open charm system, $cq\bar{q}\bar{q}$, the interaction between the light quarks q and the c quark is suppressed in the heavy quark limit [43]. Thus, in the first approximation, three light quarks are decoupled from the heavy quark and there can be considered states compounded by u , d and s as triquarks or color nonsinglet baryons (in the bound state) [43]. Symmetry of their total wave functions is determined as in the case of fermions because there are three quarks in SU(3) group: $q\bar{q}\bar{q}$, where $q = u, d, s$. In the case of fermions the total wave function has to be antisymmetric, as well as the color state function in the case of all hadrons, and therefore the particles from multiplets $\bar{15}_S$ and $\bar{3}_S$ have the symmetric spin and flavor wave functions (χ_S, ν_S), while the particles from multiplets 6_A and $\bar{3}_A$ have the antisymmetric spin and flavor wave functions (χ_A, ν_A).

The calculation of FB contribution to tetraquark masses will be described in more details. First we will explain how to calculate the $\vec{\sigma}_i \vec{\sigma}_j$ products of Pauli spin matrices for each pair in the scalar system of four quarks. These products are the expected values of spin matrix elements. We use the spin operator eigenvalues for triplet and singlet states: (+1) and (-3), respectively. We also use the following values of the matrix elements $\vec{\sigma}_1 \vec{\sigma}_2$:

$$\begin{aligned} \langle \uparrow\downarrow | \vec{\sigma}_1 \vec{\sigma}_2 | \uparrow\downarrow \rangle &= -1 = \langle \downarrow\uparrow | \vec{\sigma}_1 \vec{\sigma}_2 | \downarrow\uparrow \rangle \\ \langle \uparrow\uparrow | \vec{\sigma}_1 \vec{\sigma}_2 | \uparrow\uparrow \rangle &= 1 = \langle \downarrow\downarrow | \vec{\sigma}_1 \vec{\sigma}_2 | \downarrow\downarrow \rangle \\ \langle \downarrow\uparrow | \vec{\sigma}_1 \vec{\sigma}_2 | \uparrow\downarrow \rangle &= 2 = \langle \uparrow\downarrow | \vec{\sigma}_1 \vec{\sigma}_2 | \downarrow\uparrow \rangle. \end{aligned} \quad (6)$$

In addition to that, for total spin S and its projection m_s , for symmetric state it holds:

$$\begin{aligned} |S=1, m_s=1\rangle &= |\uparrow\uparrow\rangle \\ |S=1, m_s=0\rangle &= \frac{1}{\sqrt{2}} |\uparrow\downarrow + \downarrow\uparrow\rangle \\ |S=1, m_s=-1\rangle &= |\downarrow\downarrow\rangle, \end{aligned} \quad (7)$$

from where we get $|\uparrow\downarrow + \downarrow\uparrow\rangle = \sqrt{2} |S=1, m_s=0\rangle$.

For antisymmetric state it holds:

$$|S=0, m_s=0\rangle = \frac{1}{\sqrt{2}} |\uparrow\downarrow - \downarrow\uparrow\rangle, \quad (8)$$

and combining expressions (7) and (8), we get: $|\uparrow\downarrow\rangle = \frac{1}{\sqrt{2}} (|S=1\rangle + |S=0\rangle)$ and $|\downarrow\uparrow\rangle = \frac{1}{\sqrt{2}} (|S=1\rangle - |S=0\rangle)$.

Then we calculate symmetric $\langle \chi_S | \vec{\sigma}_i \vec{\sigma}_j | \chi_S \rangle$ and antisymmetric matrix elements $\langle \chi_A | \vec{\sigma}_i \vec{\sigma}_j | \chi_A \rangle$ for the following pairs: $q_1 q_2$, $q_1 \bar{q}_3$, $q_1 \bar{q}_4$, $q_2 \bar{q}_3$, $q_2 \bar{q}_4$, $\bar{q}_3 \bar{q}_4$, and apply them to relations for spin wave functions (4) and (5). For example, for $q_1 q_2$ pair, we use only spins of the first and second particle, therefore the other two spins e.g. of the third and forth particle are used only to determine which addends are

non-zero. In case of symmetric spin wave function it leads from eq. (9) to eq. (10):

$$\begin{aligned} \langle \chi_S | \vec{\sigma}_1 \vec{\sigma}_2 | \chi_S \rangle &= \\ &= \frac{-1}{2\sqrt{3}} \langle (\uparrow_1 \downarrow_2 + \downarrow_1 \uparrow_2) \uparrow_3 \downarrow_4 + (\uparrow_1 \downarrow_2 + \downarrow_1 \uparrow_2) \downarrow_3 \uparrow_4 + \\ &\quad - 2 \uparrow_1 \uparrow_2 \downarrow_3 \downarrow_4 - 2 \downarrow_1 \downarrow_2 \uparrow_3 \uparrow_4 | \times \\ &\times (\vec{\sigma}_1 \vec{\sigma}_2) | (\uparrow_1 \downarrow_2 + \downarrow_1 \uparrow_2) \uparrow_3 \downarrow_4 + (\uparrow_1 \downarrow_2 + \downarrow_1 \uparrow_2) \downarrow_3 \uparrow_4 + \\ &\quad - 2 \uparrow_1 \uparrow_2 \downarrow_3 \downarrow_4 - 2 \downarrow_1 \downarrow_2 \uparrow_3 \uparrow_4 \rangle \frac{-1}{2\sqrt{3}}, \end{aligned} \quad (9)$$

$$\begin{aligned} \langle \chi_S | \vec{\sigma}_1 \vec{\sigma}_2 | \chi_S \rangle &= \\ &= \frac{1}{4 \cdot 3} (\underbrace{\langle (\uparrow \downarrow + \downarrow \uparrow) \rangle}_I + \underbrace{\langle (\uparrow \downarrow + \downarrow \uparrow) \rangle}_{II} - 2 \uparrow \uparrow - 2 \downarrow \downarrow | \times \\ &\times (\vec{\sigma}_1 \vec{\sigma}_2) | (\uparrow \downarrow + \downarrow \uparrow) + (\uparrow \downarrow + \downarrow \uparrow) - 2 \uparrow \uparrow - 2 \downarrow \downarrow \rangle). \end{aligned} \quad (10)$$

Nevertheless, we see that addends labeled with I and II in eq. (10) cannot be combined into non-zero matrix elements because the spins of the third and fourth particles are $\uparrow \downarrow$ in case I, and $\downarrow \uparrow$ in case II. So we derive:

$$\begin{aligned} \langle \chi_S | \vec{\sigma}_1 \vec{\sigma}_2 | \chi_S \rangle &= \\ &\frac{1}{12} (\sqrt{2} \langle S=1, m_s=0 | (\vec{\sigma}_1 \vec{\sigma}_2) | S=1, m_s=0 \rangle \sqrt{2} + \\ &+ \sqrt{2} \langle S=1, m_s=0 | (\vec{\sigma}_1 \vec{\sigma}_2) | S=1, m_s=0 \rangle \sqrt{2} + \\ &+ (-2) \langle S=1, m_s=1 | (\vec{\sigma}_1 \vec{\sigma}_2) | S=1, m_s=1 \rangle (-2) + \\ &+ (-2) \langle S=1, m_s=-1 | (\vec{\sigma}_1 \vec{\sigma}_2) | S=1, m_s=-1 \rangle (-2)), \end{aligned} \quad (11)$$

from where we obtain this result: $\langle \chi_S | \vec{\sigma}_1 \vec{\sigma}_2 | \chi_S \rangle = \frac{1}{12} (2 \cdot 1 + 2 \cdot 1 + 4 \cdot 1 + 4 \cdot 1) = 1$.

In this way we get the results for all quark pairs $q_i q_j$:

$$\begin{aligned} \langle \chi_S | \vec{\sigma}_1 \vec{\sigma}_2 | \chi_S \rangle &= 1 = \langle \chi_S | \vec{\sigma}_3 \vec{\sigma}_4 | \chi_S \rangle \\ \langle \chi_S | \vec{\sigma}_1 \vec{\sigma}_3 | \chi_S \rangle &= -2 = \langle \chi_S | \vec{\sigma}_2 \vec{\sigma}_4 | \chi_S \rangle \\ \langle \chi_A | \vec{\sigma}_1 \vec{\sigma}_2 | \chi_A \rangle &= -3 = \langle \chi_A | \vec{\sigma}_3 \vec{\sigma}_4 | \chi_A \rangle \\ \langle \chi_A | \vec{\sigma}_1 \vec{\sigma}_3 | \chi_A \rangle &= 0 = \langle \chi_A | \vec{\sigma}_2 \vec{\sigma}_4 | \chi_A \rangle. \end{aligned} \quad (12)$$

Table 1.

The products $\lambda_i \lambda_j$ (Gell-Mann matrices for color $SU(3)_C$) and the products $\vec{\sigma}_i \vec{\sigma}_j$ (Pauli spin matrices) for symmetric and antisymmetric multiplets.

multiplet	$q_i q_j, \bar{q}_i \bar{q}_j$	$q_i \bar{q}_j$
$\bar{15}_S$ and $\bar{3}_S$	$\lambda_1 \lambda_2 = \lambda_3 \lambda_4 = -\frac{8}{3}$ $\vec{\sigma}_1 \vec{\sigma}_2 = \vec{\sigma}_3 \vec{\sigma}_4 = 1$	$\lambda_1 \lambda_3 = \lambda_1 \lambda_4 = \lambda_2 \lambda_3 = \lambda_2 \lambda_4 = -\frac{4}{3}$ $\vec{\sigma}_1 \vec{\sigma}_3 = \vec{\sigma}_1 \vec{\sigma}_4 = \vec{\sigma}_2 \vec{\sigma}_3 = \vec{\sigma}_2 \vec{\sigma}_4 = -2$
6_A and $\bar{3}_A$	$\lambda_1 \lambda_2 = \lambda_3 \lambda_4 = -\frac{8}{3}$ $\vec{\sigma}_1 \vec{\sigma}_2 = \vec{\sigma}_3 \vec{\sigma}_4 = -3$	$\lambda_1 \lambda_3 = \lambda_1 \lambda_4 = \lambda_2 \lambda_3 = \lambda_2 \lambda_4 = -\frac{4}{3}$ $\vec{\sigma}_1 \vec{\sigma}_3 = \vec{\sigma}_1 \vec{\sigma}_4 = \vec{\sigma}_2 \vec{\sigma}_3 = \vec{\sigma}_2 \vec{\sigma}_4 = 0$

In Table 1 we give the products of Gell-Mann matrices for color $SU(3)_C$ and the products of Pauli spin matrices. When we put values for $\vec{\sigma}_i \vec{\sigma}_j$ and $\lambda_i \lambda_j$ from Table 1 into equation (2), we get the following FB HFI contributions for symmetric

Table 2.

Masses of scalar $cq\bar{q}\bar{q}$ tetraquarks distributed in $SU(3)_F$ multiplets, with mixing between states with the same quantum numbers. $m_{\nu,0}$ are tetraquark masses without influence of FB HFI and $m_{\nu,FB}$ are FB HFI contributions to tetraquark masses.

multiplet	$cq\bar{q}\bar{q}$	$m_{\nu,0} (m_u = m_d)$	$m_{\nu,FB} (m_u = m_d)$
$\bar{15}_S$	Ξ	$m_u + 2m_s + m_c$	$\frac{8}{3}C \left(\frac{2}{m_s m_c} + \frac{2}{m_u m_s} - \frac{1}{m_u m_c} - \frac{1}{m_s^2} \right)$
	Σ_s	$2m_u + m_s + m_c$	$\frac{8}{3}C \left(\frac{1}{m_u^2} + \frac{1}{m_s m_c} \right)$
	Δ	$3m_u + m_c$	$\frac{8}{3}C \left(\frac{1}{m_u^2} + \frac{1}{m_u m_c} \right)$
	Σ	$2m_u + m_s + m_c$	$\frac{8}{3}C \left(\frac{2}{m_u m_c} + \frac{2}{m_u m_s} - \frac{1}{m_s m_c} - \frac{1}{m_u^2} \right)$
$\bar{15}_S - \bar{3}_S$	D_s	$2m_u + m_s + m_c; 3m_s + m_c$	$\frac{8}{3}C \left(\frac{1}{m_u^2} + \frac{1}{m_s m_c} \right); \frac{8}{3}C \left(\frac{1}{m_s^2} + \frac{1}{m_s m_c} \right)$
	D	$3m_u + m_c; m_u + 2m_s + m_c$	$\frac{8}{3}C \left(\frac{1}{m_u^2} + \frac{1}{m_u m_c} \right); \frac{4}{3}C \left(\frac{3}{m_s^2} + \frac{2}{m_u m_c} - \frac{1}{m_u^2} \right)$
6_A	Σ_s	$2m_u + m_s + m_c$	$8C \left(\frac{1}{m_u m_s} + \frac{1}{m_u m_c} \right)$
	Ω	$2m_u + m_s + m_c$	$8C \left(\frac{1}{m_u^2} + \frac{1}{m_s m_c} \right)$
$\bar{3}_A$	D_s	$2m_u + m_s + m_c$	$8C \left(\frac{1}{m_u m_s} + \frac{1}{m_u m_c} \right)$
$6_A - \bar{3}_A$	D	$3m_u + m_c; m_u + 2m_s + m_c$	$8C \left(\frac{1}{m_u m_s} + \frac{1}{m_s m_c} \right); 8C \left(\frac{1}{m_u^2} + \frac{1}{m_u m_c} \right)$

and antisymmetric multiplets:

$$m_{\nu,FB,S} = -\frac{8}{3}C \langle \nu_S | \frac{1}{m_1 m_2} + \frac{1}{m_3 m_4} | \nu_S \rangle + \frac{8}{3}C \langle \nu_S | \frac{1}{m_1 m_3} + \frac{1}{m_1 m_4} + \frac{1}{m_2 m_3} + \frac{1}{m_2 m_4} | \nu_S \rangle, \quad (13)$$

for symmetric multiplets $\bar{15}_S$ and $\bar{3}_S$, and

$$m_{\nu,FB,A} = 8C \langle \nu_A | \frac{1}{m_1 m_2} + \frac{1}{m_3 m_4} | \nu_A \rangle, \quad (14)$$

for antisymmetric multiplets 6_A and $\bar{3}_A$. The flavor wave functions ν_S and ν_A for the scalar $cq\bar{q}\bar{q}$ tetraquarks are given in Table II of Ref. [41]. The masses of tetraquarks predicted from our model are given in Table 2. The mixing between states with the same quantum numbers is included. There is mixing between states from symmetric multiplets $\bar{15}_S$ and $\bar{3}_S$, and also between antisymmetric multiplets 6_A and $\bar{3}_A$, while symmetric and antisymmetric multiplets do not mix with each other. We first show

our predictions for spectra when FB HFI is not included (the third column of the table) and then we show the FB HFI influence (the forth column).

Here we study mass spectra of single-charm tetraquarks using FB HFI in schematic approximation (two-particle interaction). The masses of constituent quarks m_u ($= m_d$), m_s and m_c and the constant are calculated from χ^2 fitting of the equations for meson and for baryon masses, with FB interaction included, to the experimental meson and baryon masses [36]. As one can see from Table 2 in Ref. [44] or Table 2 in Ref. [45] and from references therein, our predictions for constituent quark masses are similar to masses obtained using different phenomenological models. We use masses for these mesons: light pseudoscalar mesons π , K , light vector mesons ρ , K^* , ω , φ , charmed mesons D , D^* and strange charmed mesons D_S , D_S^* . We did not calculate η and η' contribution because of their mixing and because they cannot be described within such a model. Also, we use masses for these baryons: light baryon octet N , Σ , Ξ , Λ , light baryon decuplet Δ , Σ^* , Ξ^* , Ω and heavy baryons Σ_c , Λ_c , Σ_c^* , Ω_c . For each set of equations, the minimized χ^2 values for masses are calculated by formula (14) given in Ref. [41]. The corresponding experimental masses are taken from the "Particle Data Group" site: <http://pdg.lbl.gov> [46].

From the χ^2 fit of all meson masses (when mesons with two c -quarks are excluded) we obtained the following values [36] (Table III): $m_u = m_d = 314.75$ MeV, $m_s = 466.80$ MeV, $m_c = 1627.31$ MeV and the constant $C^m = 1.5546 \times 10^7 \text{ MeV}^3$. These values for quark masses here we use for calculating the tetraquark masses. For the constant C we use value C^{tetra} which is different from C^{meson} . We fitted C^{tetra} in that way to obtain the mass of the lowest state from $\bar{3}_A$ equal as the $D_s^+(2317)$ meson: $C^{tetra} = -5.80 \times 10^6 \text{ MeV}^3$. From the fit of all baryon masses (when barions with two c -quarks are excluded), in [36] (Table III) we obtained: $m_u = m_d = 365.69$ MeV, $m_s = 530.08$ MeV, $m_c = 1700.17$ MeV and the constant $C^b = 1.2513 \times 10^7 \text{ MeV}^3$. Here, we calculated $C^{tetra} = -11.90 \times 10^6 \text{ MeV}^3$ (in that way to obtain the mass of the lowest state from $\bar{3}_A$ equal as the $D_s^+(2317)$ meson).

The values of theoretical masses of some mesons and baryons with FB HFI included, using constituent quark masses and the constants C^m , C^b given in Table III of Ref. [36] (the upper rows which correspond to FB HFI) we present in Tables 3 and 4. If we compare Tables 3 and 4 we can notice that theoretically obtained masses of baryons using FB HFI are in better agreement with the corresponding experimental masses than masses of mesons.

The values of masses (in MeV) of scalar $cq\bar{q}\bar{q}$ tetraquarks, obtained from the meson fit, are given in Table 5, and their masses obtained from the baryon fit are given in Table 6. These tables with tetraquark masses show us where the 27 tetraquark states are expected. There are uncertainties in calculating masses because we use the model with schematic interaction.

The tetraquark mass spectra are given in Figs. 1 and 2 (labels are the same as

Table 3.

The values of theoretical masses m_m (in MeV) of some mesons with FB HFI included, when we use constituent quark masses and the constant C^m obtained from the meson fit (Table III of Ref. [36]). m_m^{exp} are experimental masses [46] and Δm_m is the absolute difference between these two values.

meson	m_m (MeV)	m_m^{exp} (MeV)	Δm_m (MeV)
π	159	140	19
K	464	494	30
ρ	786	776	10
K^*	887	892	5
ω	786	783	3
φ	1005	1020	15
D	1851	1869	18
D^*	1972	2010	38
D_S	2033	1968	65
D_S^*	2015	2012	3

Table 4.

The values of theoretical masses m_b (in MeV) of some baryons with FB HFI included, when we use constituent quark masses and the constant C^b obtained from the baryon fit (Table III of Ref. [36]). m_b^{exp} are experimental masses [46] and Δm_b is the absolute difference between these two values.

baryon	m_b (MeV)	m_b^{exp} (MeV)	Δm_b (MeV)
N	957	940	17
Σ	1179	1190	11
Ξ	1319	1315	4
Λ	1121	1116	5
Δ	1237	1232	5
Σ^*	1373	1385	12
Ξ^*	1513	1530	17
Ω	1657	1672	15
Σ_c	2438	2455	17
Λ_c	2291	2285	6

in Ref. [41]). It can be noticed that HFI determines mass splitting in the spectrum, i.e. its fine structure. And for mixing it can be said that it separates two states. When comparing Fig. 1 (or Fig. 2) from this paper with Fig. 6 from [41] and Fig. 1 from [42], it can be noticed that GR HFI reduces masses (except for $\overline{15}_S - \overline{3}_S$ mixed states) more than FB HFI, but that difference is not so significant. One can see that

Table 5.

The values of masses (in MeV) of scalar $cq\bar{q}\bar{q}$ tetraquarks distributed in $SU(3)_F$ multiplets, with mixing between states with the same quantum numbers, obtained from the meson fit. $m_{\nu,0}$ (MeV) are tetraquark masses without influence of FB HFI, $m_{\nu,FB}$ (MeV) are FB HFI contributions to tetraquark masses and m_ν (MeV) are the total tetraquark masses.

multiplet	tetraquark	$m_{\nu,0}$ (MeV)	$m_{\nu,FB}$ (MeV)	m_ν (MeV)
$\bar{15}_S$	Ξ	2876	-150	2726
	Σ_s	2724	-176	2547
	Δ	2572	-186	2385
	Σ	2724	-94	2629
$\bar{15}_S - \bar{3}_S$	$D_s(\bar{15}_S - \bar{3}_S)$	2724; 3028	-176; -91	2547; 2936
	$D(\bar{15}_S - \bar{3}_S)$	2572; 2876	-186; -59	2385; 2817
6_A	Σ_s	2724	-406	2317
	Ω	2724	-529	2194
$\bar{3}_A$	D_s	2724	-406	2317
$6_A - \bar{3}_A$	$D(6_A - \bar{3}_A)$	2572; 2876	-377; -559	2195; 2317

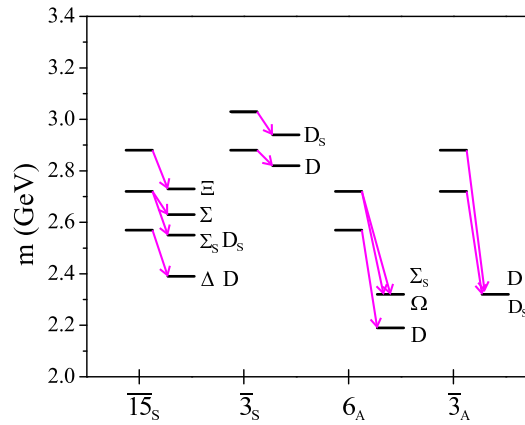


Fig. 1 – Tetraquark mass spectrum from the meson fit without (left column) and with (right column) FB HFI, both with $SU(3)_F$ symmetry breaking.

FB HFI reduces the obtained masses and causes splitting between Σ and Σ_s (D_s) in $\bar{15}_S$ and between Σ_s and Ω in 6_A , and GR HFI only causes splitting between Σ_s and Ω . Also, there is difference for $\bar{15}_S - \bar{3}_S$ mixed states when comparing with GR

Table 6.

The values of masses (in MeV) of scalar $cq\bar{q}\bar{q}$ tetraquarks distributed in $SU(3)_F$ multiplets, with mixing between states with the same quantum numbers, obtained from the baryon fit. $m_{\nu,0}$ (MeV) are tetraquark masses without influence of FB HFI, $m_{\nu,FB}$ (MeV) are FB HFI contributions to tetraquark masses and m_{ν} (MeV) are the total tetraquark masses.

multiplet	tetraquark	$m_{\nu,0}$ (MeV)	$m_{\nu,FB}$ (MeV)	m_{ν} (MeV)
$\bar{15}_S$	Ξ	3126	-234	2892
	Σ_s	2962	-273	2689
	Δ	2797	-288	2509
	Σ	2962	-157	2805
$\bar{15}_S - \bar{3}_S$	$D_s(\bar{15}_S - \bar{3}_S)$	2962; 3290	-273; -148	2689; 3142
	$D(\bar{15}_S - \bar{3}_S)$	2797; 3126	-288; -102	2509; 3024
6_A	Σ_s	2962	-644	2317
	Ω	2962	-818	2144
$\bar{3}_A$	D_s	2962	-644	2317
$6_A - \bar{3}_A$	$D(6_A - \bar{3}_A)$	2797; 3126	-597; -865	2200; 2261

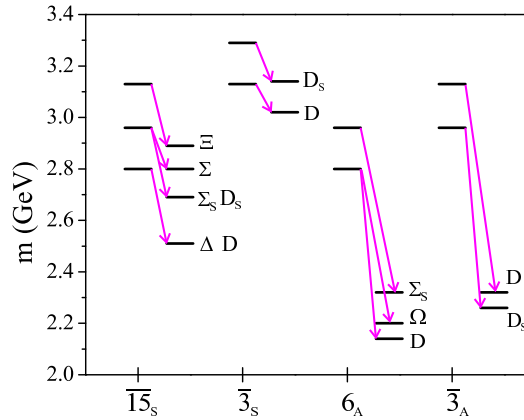


Fig. 2 – the same as Fig. 1, but for baryon fit.

HFI, but the forms of tetraquark spectra with FB and GR interactions are similar. It is interesting to note that the spectra are similar although the one interaction is color-spin, and the other one is flavor-spin.

From spectrum it is possible to identify $D_s^+(2317)$ as the lowest state in multi-

plet $\bar{3}_A$. It agrees with the identification of $D_s^+(2317)$ with tetraquark state which was considered in Ref. [47]. In Ref. [47], for $D_s^+(2632)$ it was claimed to be a candidate for a tetraquark D_s , but they proposed different total angular momentum ($J^P = 2^+$). In the present paper, the authors consider scalar tetraquark only. Also, from spectrum we can see that FB HFI reduces the obtained masses for all states.

States $D_s^+(2632)$ [8] and $D^0(2308)$ [6] are mixed states: $D_s^+(2632)$ is from $\bar{15}_S - \bar{3}_S$ and $D^0(2308)$ from $6_A - \bar{3}_A$ mixing. Because of that mixing, their flavor wave functions are given only in a first approximation [1] and therefore calculations are not sufficiently precise. Therefore these two states have theoretical predictions which are not the same as the experimental ones. But nevertheless, for meson fit, their experimental masses are still between the two values we obtained in Table 5: $D_s^+(2632)$ has the mass between 2547 and 2936 MeV, while $D^0(2308)$ has the mass between 2195 and 2317 MeV. Obtained theoretical masses for baryon fit in case of $D^0(2308)$ is lower than it is expected and for $D_s^+(2632)$ is approximately in the expected range. If we compare Tables 5 and 6 and Figures 1 and 2 we can conclude that FB interaction gives lower total masses. FB interaction that we applied needs some further improvement. Probably the biggest discrepancy between theoretically obtained and experimentally detected state is because of sensitivity of constituent quark masses to the system in which they are contained (meson, baryon, tetraquark).

4. CONCLUSIONS

We calculated the mass spectra of two-quark-two-antiquark system. We used the observed meson masses, taken from "Particle Data Group" [46], to obtain constituent quark masses by way of χ^2 mass fit. Applying the Hamiltonian (1) to the constituent quarks, we obtained the theoretical meson, baryon and single-charm tetraquark masses with FB contribution included. For the first time the FB HFI, which is color-spin interaction, is applied to 27 states of $cq\bar{q}\bar{q}$ tetraquarks in order to derive formulas for masses and these are new results. Also, when comparing with GR HFI, which is flavor-spin interaction, it is interesting that we also obtained similar results for tetraquark masses, like in the case of meson and baryon masses. In our constituent quark model, FB HFI is calibrated in mesons and baryons, and then used in tetraquarks.

If the tetraquark states we have studied contained no heavy quarks, their mass would be too low; if they contained one heavy quark (b or t) or two c -quarks, mass would be too high. According to that and to the mass constraint of the experimentally indicated states, these states can contain no more than one c -quark. We also showed [36] that the constituent quark masses are very sensitive to the system in which they are contained, and their values differ less or more in different systems.

That is why we analyzed tetraquark masses using meson and baryon fits for constituent quark masses, independently. Constituent quark masses in tetraquarks are somewhat different for both fits. We choose relatively simple FB HFI because it nicely satisfy Roncaglia inequalities for mass differences [37, 38].

Symmetric $\overline{15}_S$ -plet mixes with the $\overline{3}_S$ -plet ideally (D_s and D states). This mixing splits the two states into a heavy (hidden strangeness) and a light one. Also, antisymmetric 6_A -plet mixes with $\overline{3}_A$ -plet (lowest mass D is the ideal mixture of these antisymmetric multiplets, while the lowest D_s is pure $\overline{3}_A$ -plet). From the spectra of single-charm tetraquark masses with Fermi-Breit HFI, it can be noticed that this interaction implies no flavor dependent splitting among multiplets.

Flavor wave functions of the mixed states are given only in a first approximation (see Ref. [1]). The mixing of the states also changes the properties and shifts masses from the theoretical predictions. For instance, possibly tetraquark states $D^0(2308)$ and $D_s^+(2632)$ in our case are mixed states and the calculations of their masses are not sufficiently precise. $D_s^+(2632)$ appears as a mixed state from mixing of multiplets $\overline{15}_S$ and $\overline{3}_S$, and $D^0(2308)$ would be from mixing 6_A and $\overline{3}_A$. According to our results, all three states $D_s^+(2632)$, $D^0(2308)$, $D_s^+(2317)$ might have the tetraquark nature. We gave the contribution of FB HFI to $cq\bar{q}\bar{q}$ tetraquark masses and also we calculated tetraquark masses and compared them to GR HFI.

As it can be seen from Tables 5 and 6 and Figs. 1 and 2, FB HFI reduces the obtained masses and causes splitting between Σ and Σ_s in $\overline{15}_S$ and between Σ_s and Ω in 6_A . Besides, the spectra obtained from different fits have a similar arrangement of particles. Probably the biggest difference between theoretical and experimental states is due to:

- (i) sensitivity of constituent quark masses on systems in which they are contained,
- (ii) wave functions for the two detected experimental states $D^0(2308)$ and $D_s^+(2632)$ are calculated only in a first approximation, and obtained masses are not precise,
- (iii) FB HFI is not the completed HFI.

More experimental searches for detection of other $cq\bar{q}\bar{q}$ members are needed in the future.

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